Hardware & Software Verification

John Wickerson & Pete Harrod

Lecture 6

Last lecture

- We need to be able to **reason about** the programs we write, not merely **test** them. There is a large and growing need for this.
- Dafny is a **verification-oriented** programming language. Its compiler will refuse to produce executable code until it has proven the code to be **correct**.

But what does correct mean?

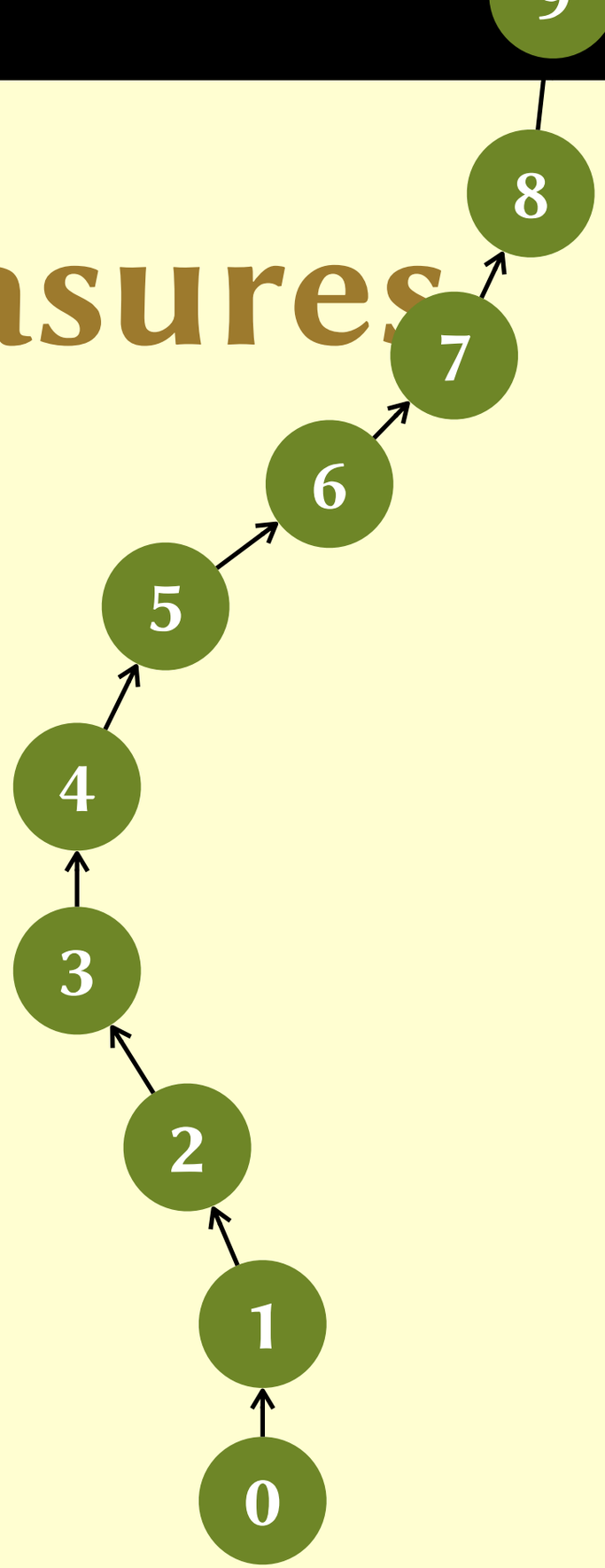
Demo: max of a pair

- named output parameters
- postconditions
- overly weak/strong specifications

Demo: max of an array

Termination measures

- A *measure* is an expression that evaluates to a non-negative integer.
- The measure must *strictly decrease* every time we go round the loop.
- Hence we can't go round the loop forever!
- E.g.: `A.Length - i`
- "Theory of well-founded relations"



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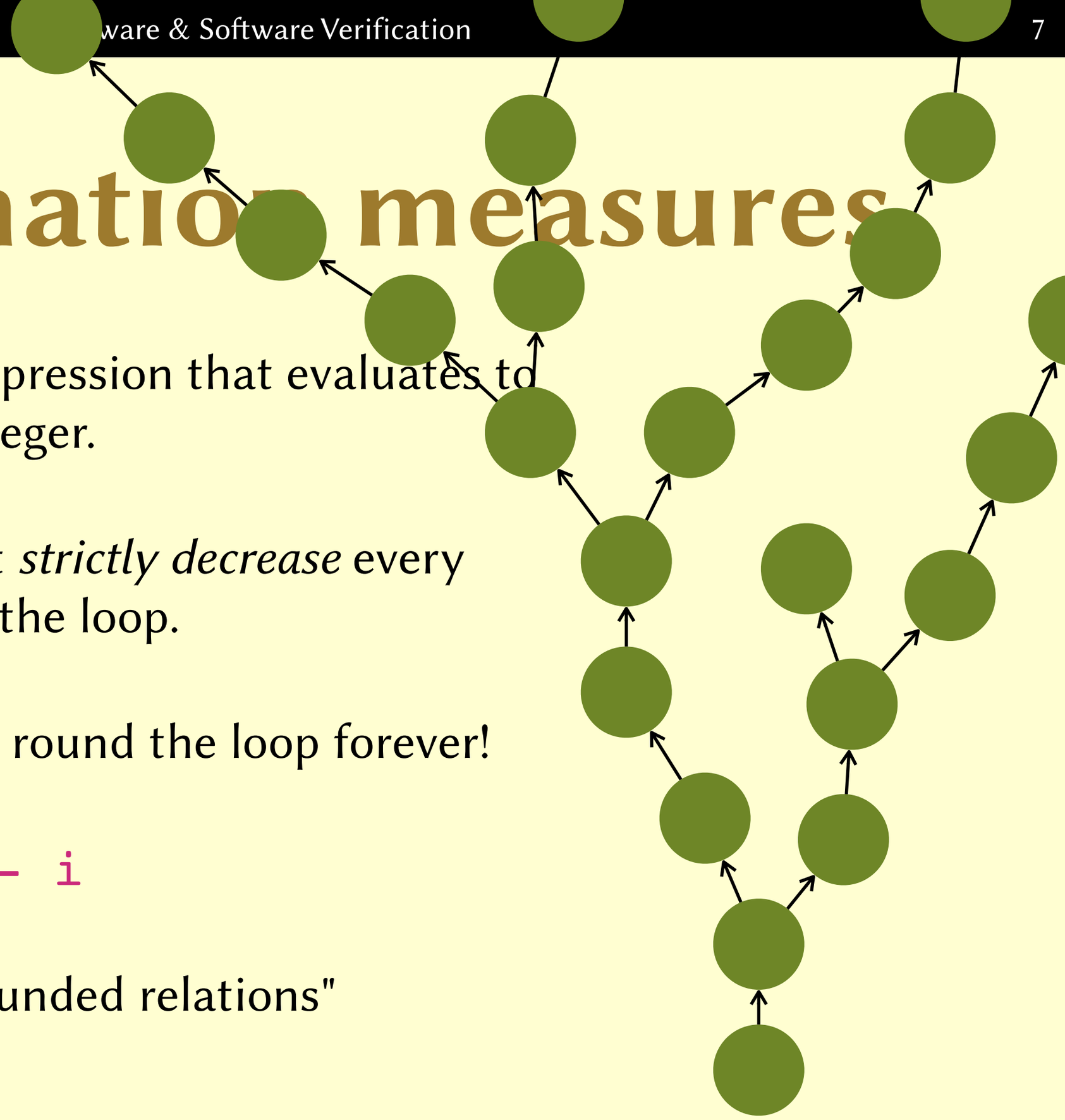
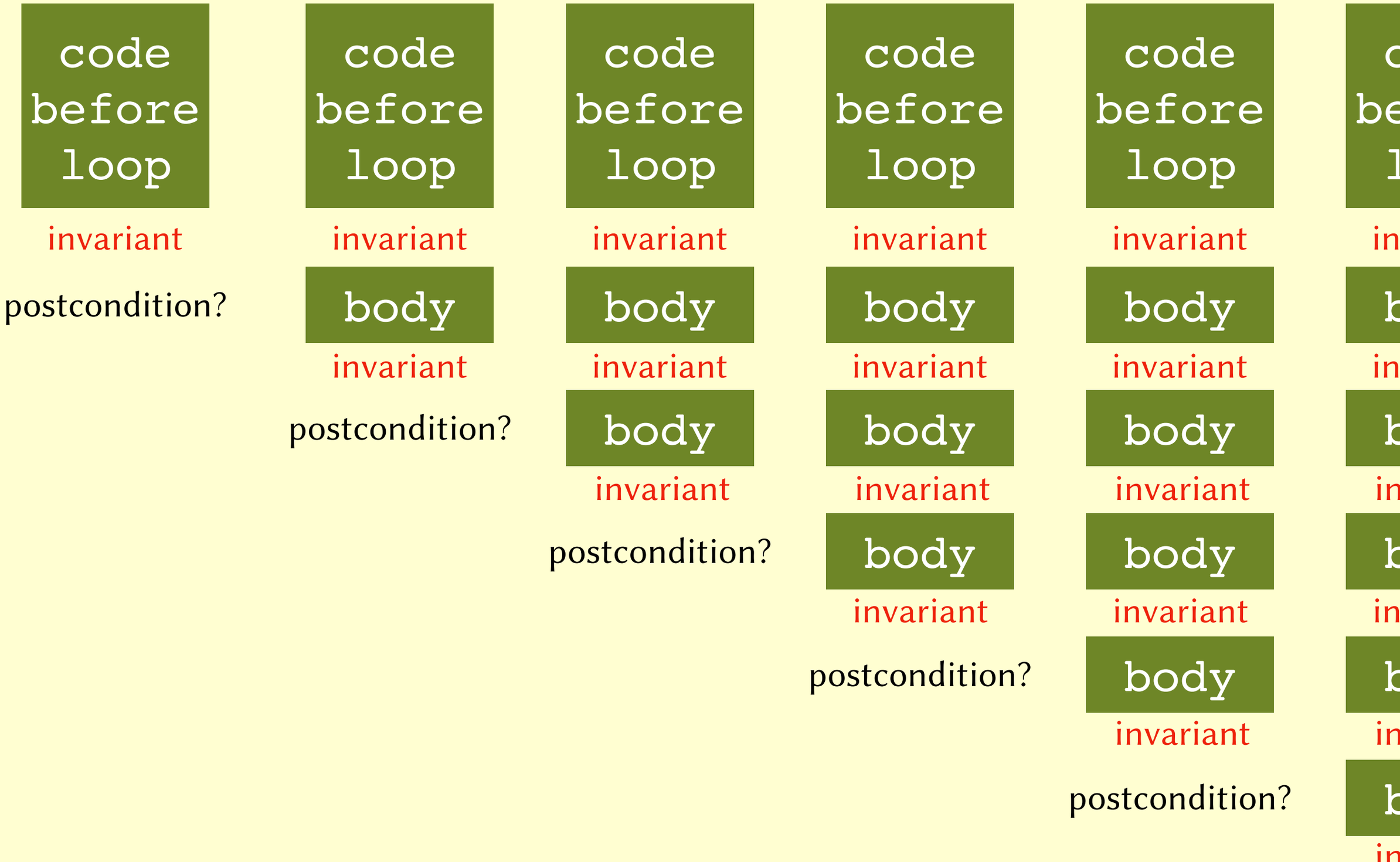


Demo: max of an array

The problem with loops



Loop invariants

1.

```
code
before
loop
```

⇓

invariant

2.

invariant

```
body
```

⇓

invariant

3.

invariant

⇓

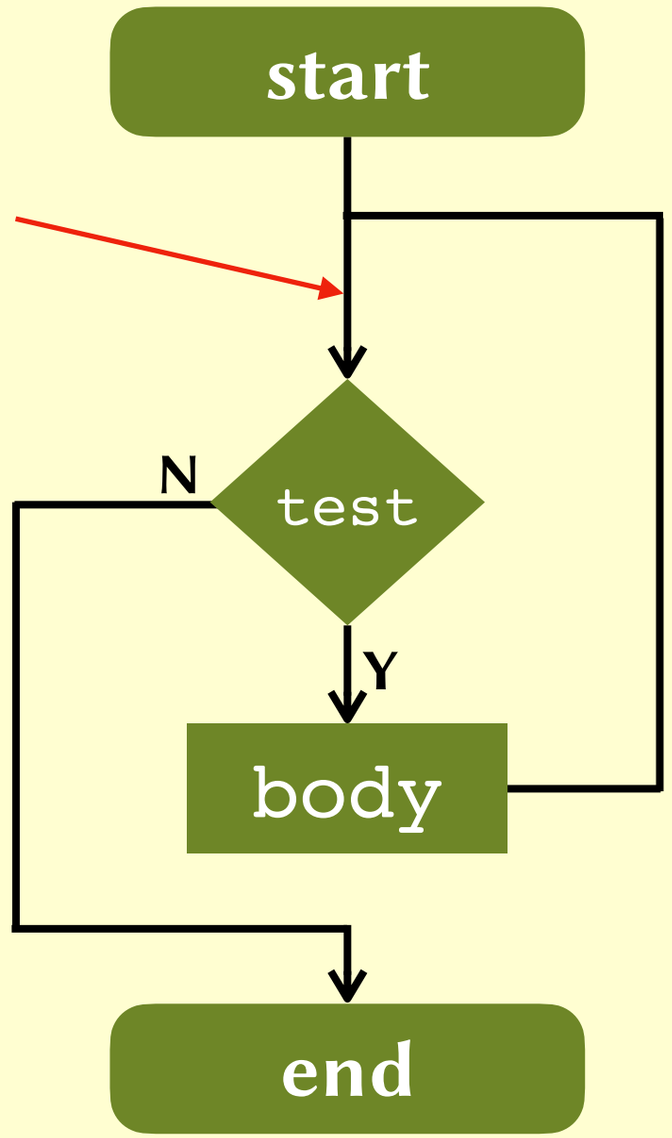
postcondition

Demo: max of an array

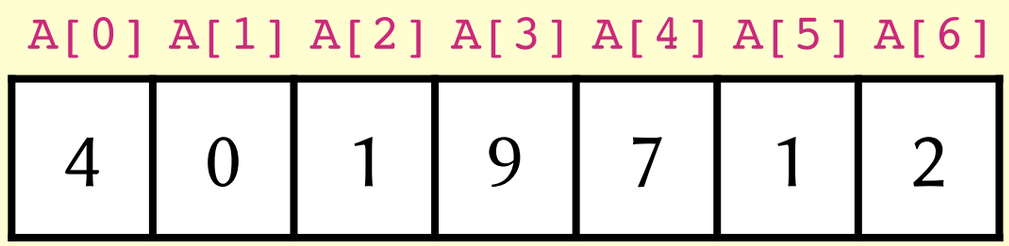
Loop invariants

```
while test
   invariant foo
{
   body
}
```

foo must hold here!



Finding invariants

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r := A[0];
var i := 1;
while i < A.Length {
   if r < A[i] {
      r := A[i];
   }
   i := i+1;
}
```

i	r
1	4
2	4
3	4
4	9
5	9
6	9
7	9

Finding invariants

A[0]	A[1]	A[2]	A[3]	A[4]	A[5]	A[6]
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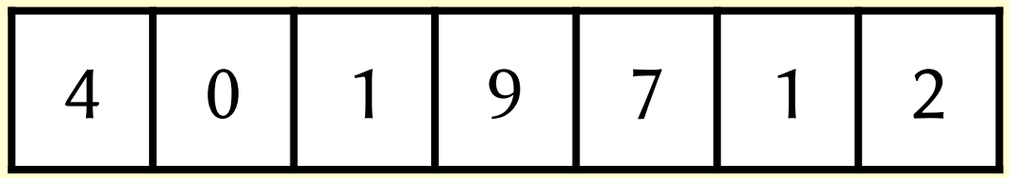


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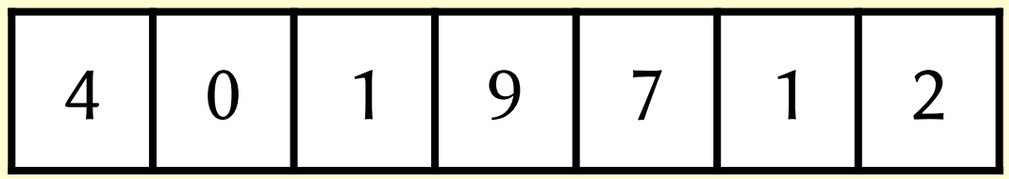


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Demo: max of an array

- syntax for variables (`var`) and arrays (`array<...>`)
- preconditions (`requires`)
- termination measures (`decreases`)
- universal (`forall`) and existential (`exists`) quantification
- loop invariants (`invariant`)
- predicates (`predicate`)

Coursework 1

- Worksheet is now on Github!
- All coursework is due **Friday 16th December at 23:59**.
- Please submit a single Dafny source file via Teams.
- Please include lots of `/*comments*/` in your source file to explain your thinking.
- Please work in pairs, and submit one file per pair.
- Please do not share your answers with other pairs.
- If you have questions, please come to the Monday labs or raise an issue on Github.