Hardware & Software Verification

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Lecture 5: More Isabelle

Lecture Outline

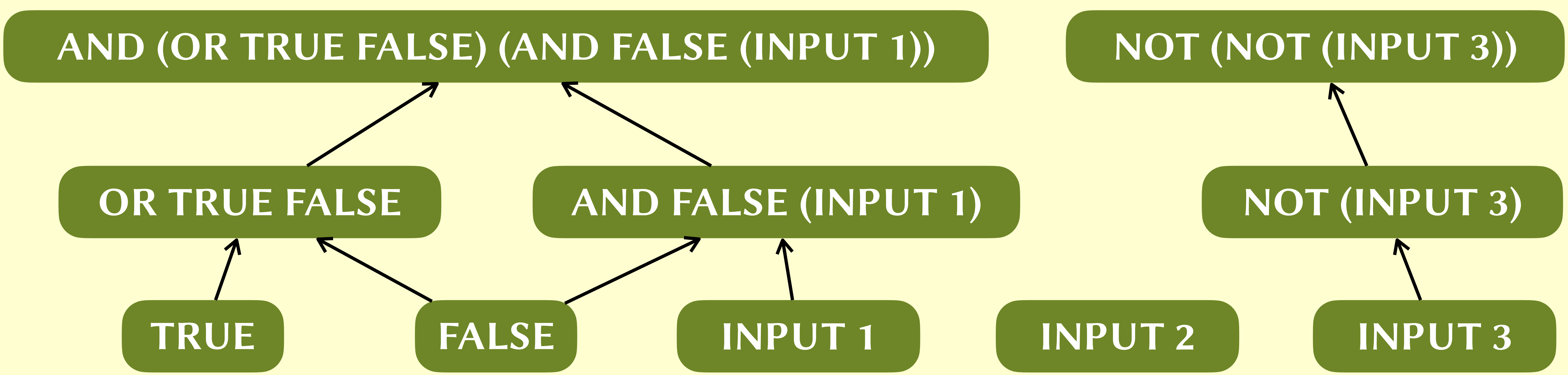
- Proving the correctness of a logic synthesiser.


DEMO

Recursive data structures

```
datatype "circuit" =
  NOT "circuit"
| AND "circuit" "circuit"
| OR "circuit" "circuit"
| TRUE
| FALSE
| INPUT "int"
```

circuit ::= NOT *circuit*
| AND *circuit circuit*
| OR *circuit circuit*
| TRUE
| FALSE
| INPUT *int*



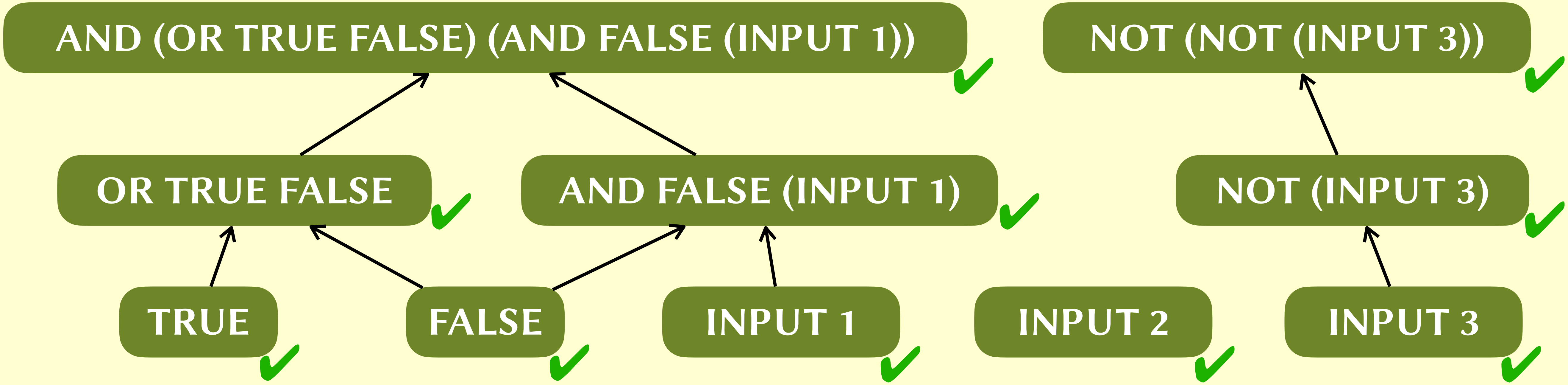

DEMO

Structural induction

- Suppose we want to show that property P holds for all circuits.

- It suffices to show that each constructor preserves P.

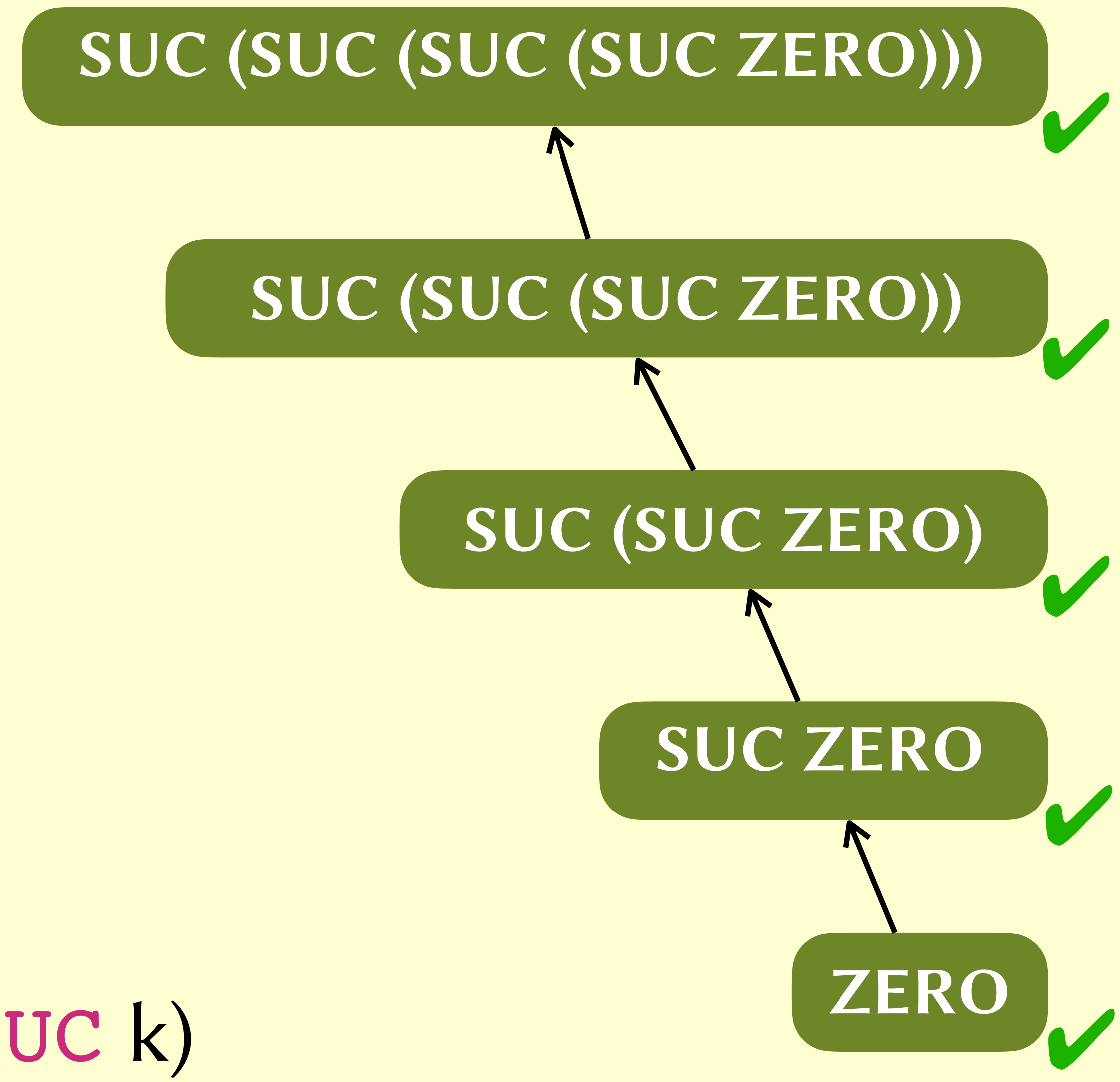
1. $\forall c.\ P(c) \Rightarrow P(\texttt{NOT}\ c)$
2. $\forall c_1,c_2.\ (P(c_1) \wedge P(c_2)) \Rightarrow P(\texttt{AND}\ c_1\ c_2)$
3. $\forall c_1,c_2.\ (P(c_1) \wedge P(c_2)) \Rightarrow P(\texttt{OR}\ c_1\ c_2)$
4. $P(\texttt{TRUE})$
5. $P(\texttt{FALSE})$
6. $\forall i.\ P(\texttt{INPUT}\ i)$



Mathematical induction

```
datatype "nat" =
   ZERO
|  SUC "nat"
```

$$\mathit{nat} ::= \texttt{ZERO} \mid \texttt{SUC}\ \mathit{nat}$$



1. $P(\texttt{ZERO})$
2. $\forall k.\ P(k) \Longrightarrow P(\texttt{SUC}\ k)$

Proof by structural induction

- **Theorem.** simulate (mirror c) ρ = simulate c ρ.
- **Proof.** We proceed by induction on the structure of c.
  - *Case "NOT":* Fix arbitrary k and assume
    simulate (mirror k) ρ = simulate k ρ
    as our induction hypothesis. We must prove that
    simulate (mirror (NOT k)) ρ = simulate (NOT k) ρ
    which we do as follows:
    simulate (mirror (NOT k)) ρ
    = simulate (NOT (mirror k)) ρ [by definition of mirror]
    = ¬ simulate (mirror k) ρ [by definition of simulate]
    = ¬ simulate k ρ [using induction hypothesis]
    = simulate (NOT k) ρ [by definition of simulate]


DEMO

Rule induction

```
fun f where
  "f (Suc (Suc n)) = f n + f (Suc n)"
| "f (Suc 0) = 1"
| "f 0 = 1"
```

- **Theorem.** f(n) ≥ n.
- **Proof.** Define P(n) = (f(n) ≥ n).
  Rule induction here requires us to prove:
  1. $\forall n.\ (P(n) \wedge P(\texttt{Suc}\ n)) \Longrightarrow P(\texttt{Suc}\ (\texttt{Suc}\ n))$
  2. $P(\texttt{Suc}\ 0)$
  3. $P(0)$

Summary

- Semantics of logical implication
- Recursive data structures
- Recursive functions
- Structural induction
- Rule induction